Readers' Forum

Brief discussions of previous investigations in the aerospace sciences and technical comments on papers published in the AIAA Journal *are presented in this special department. Entries must be restricted to a maximum of 1000 words, or the equivalent of one* Journal *page including formulas and figures. A Discussion will be published as quickly as possible after receipt of the manuscript. Neither the AIAA nor its editors are responsible for the opinions expressed by the correspondents. Authors will be invited to reply promptly.*

Comment on "The Influence of Acceleration on Laminar Similar Boundary Layers"

A. Wortman*
ISTAR Inc., Santa Monica, California

REFERENCE 1 presents a parametric study of compressible, self-similar boundary layers, employing the assumption of a constant density-viscosity product and Prandtl number of unity. The pressure gradient parameter β ranges up to 1000, wall/total enthalpy ratios range from 0.2 to 1, and elaborate correlations are presented on the basis of the claimed utility of the computed results.

It is the purpose of this Comment to point out that the highly simplified fluid properties used in Ref. 1 lead to irrelevant results, even with the limited utility of self-similar solutions. Further, the range of pressure parameters exceeds, by orders of magnitude, the values that could be found in actual engineering systems.

In a comprehensive parametric study aimed at illustrating the critical influence of fluid properties on highly accelerated boundary layers, Ref. 2 studied self-similar boundary layers with β ranging up to 20. It should be noted that at spherical and cylindrical stagnation points, the values of β are 0.5 and 1.0, respectively. Because of boundary-layer effects, even sharp junctions on hypersonic missiles indicate β's of about 5. Similarly, extremely short rocket nozzle throats have β's of the same order of magnitude. Calculations for values of β up to 1000 are therefore most unrealistic, and the use of $Pr=1$ and linear viscosity variation with temperature are unnecessary and inaccurate simplifications.

As an example of the critical influence of fluid properties on the surface shear stress parameter, Fig. 1 (Fig. 3a in Ref. 2) shows the influence viscosity-temperature relations for a range of β. In this figure, E is the Eckert number $u^2_e/(2H_e)$ and ω is the exponent in the viscosity-enthalpy relationship. The other terms are as defined in Ref. 1. Similar significant differences between the simplified case of $Pr=1$, $\omega=1$ used in Ref. 1 and more realistic variations are shown for the heat-transfer parameter in a range of β and the Mach number parameter E in Fig. 2 (Fig. 6 in Ref. 2). Any resemblance between the results of Ref. 1 and those of Ref. 2 must be viewed as coincidental.

It is now generally accepted that engineers have almost universal access to computers, and enormous gains have been made because of the enhanced computational capabilities. Demonstrations of abilities to use computers to solve relatively simple mathematical problems are no longer necessary. A redirection of the effort toward surveys of the literature and critical evaluation of the ultimate utility of the calculated results could produce more lasting contributions.

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*Technical Director. Member AIAA.

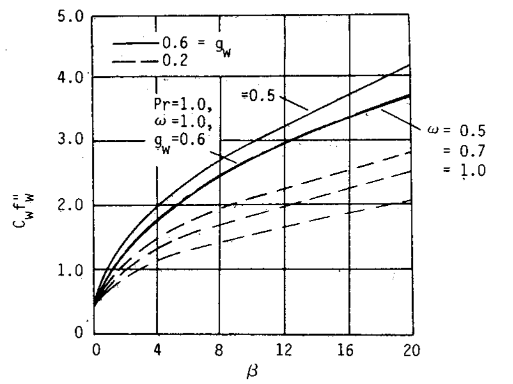

Fig. 1 Nondimensional shear stress as a function of β: effect of ω for $E=0$, $f_w=0$, $Pr=0.715$ unless otherwise specified.

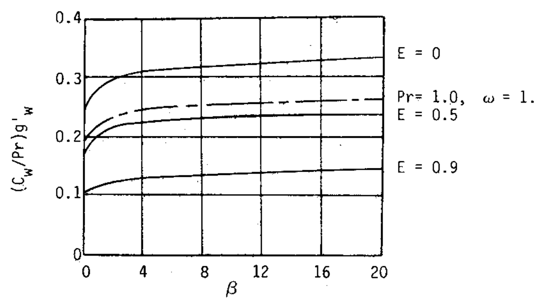

Fig. 2 Nondimensional heat-transfer rate as a function of β: effect of E for $g_w=0.6$, $f_w=0$, $Pr=0.740$, $\omega=0.5$ unless otherwise specified.

References

[1]Pade, O., Postan, A., Anshelovitz, M., and Wolfshtein, M., "The Influence of Acceleration on Laminar Similar Boundary Layers," *AIAA Journal,* Vol. 23, Oct. 1985, pp. 1469–1475.

[2]Wortman, A. and Mills, A. F., "Highly Accelerated Compressible Laminar Boundary Layer Flows with Mass Transfer," ASME Paper 70-HT/SpT-34: *Journal of Heat Transfer,* Ser. C., Vol. 93, Aug. 1971, pp. 281–289.

Reply by Authors to A. Wortman

O. Pade,* A. Postan,* and D. Anshelowitz*
Ministry of Defense, Haifa, Israel
and
M. Wolfshtein†
Technion—Israel Institute of Technology
Haifa, Israel

WORTMAN'S criticism of Pade et al.[1] may be divided into two parts: 1) the simplifications used (the model fluid and the similarity solution) are so severe that the results

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*Senior Research Scientist, Scientific Department.
†Professor, Department of Aeronautics.

lose their physical significance, and 2) the pressure gradient parameter β cannot assume in real flows such high values as discussed in Ref. 1. As these claims are based on Ref. 2, we shall recall some of the background of this paper.

General Background

The problem of compressible self-similar boundary layers is formulated by Dorrance[3] for the general case of reacting flow with mass transfer. From the analysis, it follows that the self-similar solution is possible when some parameters are constant. For the case of flow without mass transfer, two parameters are relevant:

$$E_h = (1/Pr - 1)U_e^2/H_e$$

$$\beta_0 = (2\xi/U_e)\mathrm{d}U_e/\mathrm{d}\xi$$

where ξ is the streamwise coordinate, U_e the potential velocity, and H_e the potential stagnation enthalpy. The parameter β_0 has been used extensively as the pressure gradient parameter in boundary-layer calculations and was one of the main parameters used by Wortman.[2,4] It implies a power law relation between the velocity and the streamwise coordinate ξ. The first parameter E_h is related to the Mach number. It vanishes if the Prandtl number is unity or at a stagnation point. Otherwise, it is constant only if the stagnation enthalpy is proportional to the velocity squared, which may happen if chemical reaction occurs. The chance for this to happen is small, and the case of zero E_h is the more interesting one.

Wortman solved the equations for values of β between 0 and 20: E_h of 0.0, 0.5, 0.7, and 1.0; Pr of 0.7 and 1.0; and ω (the power in the viscosity temperature law) of 0.5, 0.7, and 1.0. His results depict the importance of these parameters.

In particular, the results of Ref. 2 suggest that the influence of the Prandtl number is relatively low. The influence of ω is more significant, but for $E=0$, which corresponds to the present case, the difference is about 18%. In all the figures, the influence of ω and Pr tend to decrease as E decreases. Another interesting feature is that the influence of ω and Pr is contradictory: For instance, while a reduction of ω increases the displacement thickness, a reduction of Pr increases it and, therefore, in the real case of $\omega = 0.7$, $Pr = 0.7$, the two tend to balance one another partially.

Justification for the Model Fluid

Our calculations are restricted to the more usual case of flows with constant stagnation enthalpy. Thus, they correspond to Wortman's case of $E_h = 0$. Therefore, Wortman's calculations (3) and (5), in which E_h is equal to 0.5, 0.7, and 1, are not comparable to our calculations. Wortman's calculations for zero E_h suggest that the influence of Pr and ω is sufficiently small to make our model fluid approximation of value an engineering approximation. This conclusion is borne out by our calculations as well, and we found that the similar solutions for ω and Pr of unity are within 10% of the nonsimilar solutions for ω and Pr of about 0.7.

Magnitude of the Pressure Gradient Parameter

The other point that has been raised is concerned with the practicality of high β boundary layers. We believe that such applications are practical, for instance, in regions where abrupt changes in the cross section of nozzles occur. Such situations occur in contemporary engineering applications and are of interest to designers.

References

[1] Pade, O., Postan, A., Anshelowitz, M., and Wolfshtein, M., "The Influence of Acceleration on Laminar Similar Boundary Layers," *AIAA Journal*, Vol. 23, Oct. 1985, pp. 1469–1475.

[2] Wortman, A. and Mills, A.E., "Highly Accelerated Compressible Laminar Boundary Layer Flows with Mass Transfer," *Journal of Heat Transfer*, Ser. C. Vol. 93, No. 3, 1971, pp. 281–289.

[3] Dorrance, W.H., *Viscous Hypersonic Flow*, McGraw Hill, New York, 1962.

[4] Wortman, A. and Mills, A.E., "Separating Self-Similar Laminar Boundary Layers," *AIAA Journal*, Vol. 9, Dec. 1971, pp. 2449–2451.